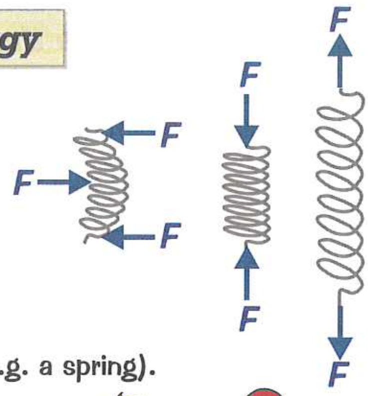
Forces and Elasticity

And now for something a bit more fun — squishing, stretching and bending stuff.

Stretching, Compressing or Bending Transfers Energy



1) When you apply a force to an object you may cause it to stretch, compress or bend.
2) To do this, you need more than one force acting on the object (otherwise the object would simply move in the direction of the applied force, instead of changing shape).
3) An object has been elastically distorted if it can go back to its original shape and length after the force has been removed.
4) Objects that can be elastically distorted are called elastic objects (e.g. a spring).
5) An object has been inelastically distorted if it doesn't return to its original shape and length after the force has been removed.
6) The elastic limit is the point where an object stops distorting elastically and begins to distort inelastically.
7) Work is done when a force stretches or compresses an object and causes energy to be transferred to the elastic potential energy store of the object. If it is elastically distorted, ALL this energy is transferred to the object's elastic potential energy store (see p.206).

Elastic objects — useful for passing exams and scaring small children

Extension is Directly Proportional to Force...

If a spring is supported at the top and then a weight is attached to the bottom, it stretches.

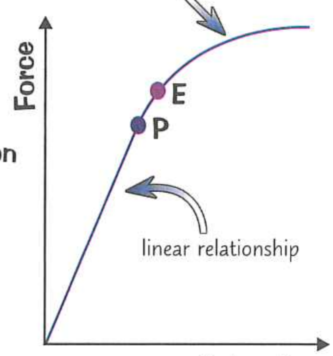
1) The extension of a stretched spring (or other elastic object) is directly proportional to the load or force applied — so $F \propto x$.
2) This means that there is a linear relationship between force and extension. (If you plotted a force-extension graph for the spring, it would be a straight line.)
3) This is the equation: $$F = k \times x$$ where F is the applied force in N, k is the spring constant in N/m and x is the extension in m.
4) The spring constant depends on the material that you are stretching — a stiffer spring has a greater spring constant.
5) The equation also works for compression (where x is just the difference between the natural and compressed lengths — the compression).

For a linear relationship, the gradient of an object's force-extension graph is equal to its spring constant.

...but this Stops Working when the Force is Great Enough

There's a limit to the amount of force you can apply to an object for the extension to keep on increasing proportionally.

1) The graph shows force against extension for an elastic object.
2) There is a maximum force above which the graph curves, showing that extension is no longer proportional to force. The relationship is now non-linear — the object stretches more for each unit increase in force. This point is known as the limit of proportionality and is shown on the graph at the point marked P.
3) The elastic limit (see above) is marked as E. Past this point, the object is permanently stretched.



I could make a joke, but I don't want to stretch myself...

That equation is pretty simple, but that doesn't mean you can skip over it. Have a go at the question below.

Q1 A spring is fixed at one end and a force of 1 N is applied to the other end, causing it to stretch. The spring extends by 2 cm. Calculate the spring constant of the spring. [2 marks]

Investigating Elasticity

You can do an easy experiment to see exactly how adding masses to a spring causes it to stretch.

You Can Investigate the Link Between Force and Extension

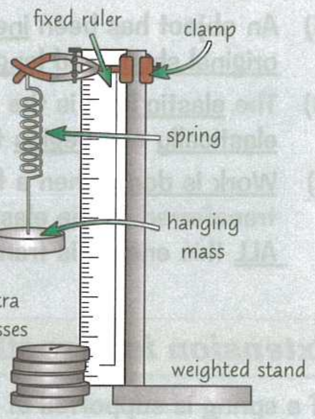
PRACTICAL

Set up the apparatus as shown in the diagram. Make sure you have plenty of extra masses, then measure the mass of each (with a mass balance) and calculate its weight (the force applied) using $W = mg$ (p.150).

You could do a quick pilot experiment first to find out what size masses to use.

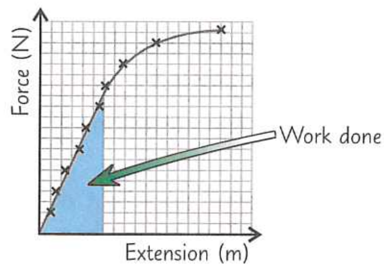
- Using an identical spring to the one you will be testing, load it with masses one at a time and record the force (weight) and extension each time.
- Plot a force-extension graph and check that you get a nice straight line for at least the first 6 points. If it curves too early, you need to use smaller masses.

1) Measure the natural length of the spring (when no load is applied) with a millimetre ruler clamped to the stand. Make sure you take the reading at eye level and add markers (e.g. thin strips of tape) to the top and bottom of the spring to make the reading more accurate.
2) Add a mass to the spring and allow the spring to come to rest. Record the mass and measure the new length of the spring. The extension is the change in length.
3) Repeat this process until you have enough measurements (no fewer than 6).
4) Plot a force-extension graph of your results. It will only start to curve if you exceed the limit of proportionality, but don't worry if yours doesn't (as long as you've got the straight line bit).



You should find that a larger force causes a bigger extension. You can also think of this as more work needing to be done to cause a larger extension. The force doing work is the gravitational force and for elastic distortions, this force is equal to $F = kx$.

You can find the work done for a particular forces (or energy stored — see below) by calculating the area under the linear section of your force-extension graph up to that value of force.



You Can Calculate Work Done for Linear Relationships

1) Look at the graph on the previous page. The elastic limit is always at or beyond the limit of proportionality. This means that for a linear relationship, the distortion is always elastic — all the energy being transferred is stored in the spring's elastic potential energy store.
2) So, as long as a spring is not stretched past its limit of proportionality, work done to the spring is equal to the energy stored in its elastic potential energy store.
3) For a linear relationship, the energy in the elastic potential energy store (and so the work done) can be found using:

$$E = \frac{1}{2} \times k \times x^2$$

Energy transferred in stretching (J); Spring constant (N/m); Extension^2 (m^2)

Time to spring into action and learn all this...

Remember that you can only use the gradient to find the spring constant if the graph is linear (a straight line).

Q1 A spring with a spring constant of 40 N/m extends elastically by 2.5 cm. Calculate the amount of energy stored in its elastic potential energy store. [2 marks]

Forces and Elasticity

1 A child is playing with a toy that contains a spring. Grade 4-6

a) Give the minimum number of forces that need to be applied to the spring in order to stretch it.

....
[1]

b) When the spring is compressed, it distorts elastically.
Explain the difference between elastic and inelastic distortion.

....

....

....
[2]

c) i) State the equation that links the force exerted on a spring, its spring constant and its extension.

....
[1]

ii) A 20 N force stretches the spring by 8 cm. Calculate the spring constant of the spring.

Spring constant = ... N/m
[2]

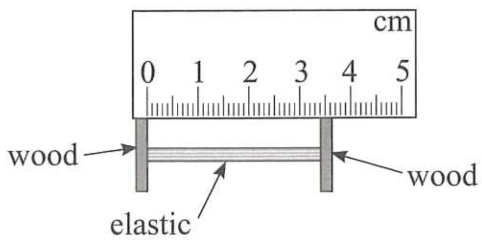
d) State **one** assumption you made to answer part c) ii).

....
[1]

[Total 7 marks]

2 **Figure 1** shows a piece of elastic being stretched between two pieces of wood. The spring constant of the elastic is 50 N/m and the unstretched length of the elastic is 3.1 cm. Grade 6-7

Figure 1



Given that the limit of proportionality hasn't been exceeded, how much energy is stored in the stretched elastic?

☐ **A** 2.89×10^{-2} J ☐ **B** 289 J ☐ **C** 2.25 J ☐ **D** 2.25×10^{-4} J

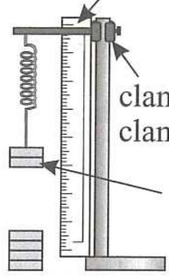
[Total 1 mark]

PRACTICAL

3 A student investigated the relationship between the extension of a spring and the forces acting on it. He hung different weights from the bottom of the spring and measured its extension with a ruler, as shown in **Figure 2**.

Grade 6–7

Figure 2

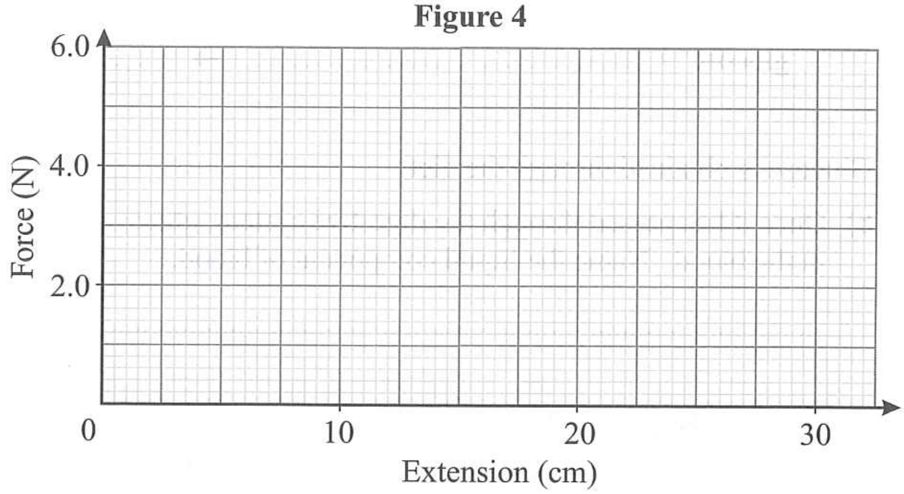


a) **Figure 3** shows the results that the student obtained in his investigation. Draw the force-extension graph for the student's results on the axes in **Figure 4**.

Figure 3

Force (N)	Extension (cm)
0.0	0.0
1.0	4.4
2.0	7.5
3.0	12.3
4.0	16.0
5.0	22.2
6.0	32.0

Figure 4



[3]

b) Using the graph you have drawn, calculate the spring constant of the spring being tested.

Spring constant = ... N/m
[2]

c) The student realised he had stretched the spring past its limit of proportionality. Explain how you can tell this from the graph.

....

....
[2]

d) The student removed the masses from the spring. Whilst he unloaded the spring, he measured its extension for each force again. He found that, when unloading the spring, the extension of the spring was 20.1 cm when a force of 4.0 N acted on it. Suggest and explain a reason for this.

....

....

....
[2]

[Total 9 marks]

Exam Practice Tip

It's easy to get caught out by problems like question 2. Remember, for the equations for stretching (or compressing), you need to use the amount the length of an object has changed by, not its total length after it's been stretched.